arXiv:0705.2204v2 [hep-ph] 15 May 2007

**Probing for Dynamics of Dark-Energy in Mass Varying Neutrinos:
Cosmic Microwave Background Radiation and Large Scale Structure**

Yong-Yeon Keum

*Department of Physics, National Taiwan University,
No.1 Sec.4, Roosevelt Road, Taipei, Taiwan, 10672, R.O.C.
yykeum@phys.ntu.edu.tw*

We present cosmological perturbation theory in neutrino probe interacting dark-energy models, and calculate cosmic microwave background anisotropies and matter power spectrum. In these models, the evolution of the mass of neutrinos is determined by the quintessence scalar field, which is responsible for the cosmic acceleration today. We consider several types of scalar field potentials and put constraints on the coupling parameter between neutrinos and dark energy. Assuming the flatness of the universe, the constraint we can derive from the current observation is $\sum m_\nu < 0.87 eV$ at the 95 % confidence level for the sum over three species of neutrinos.

Keywords: Time Varying Neutrino Masses; Neutrino Mass Bound; Cosmic Microwave Background; Large Scale Structures; Quintessence Scalar field.

PACS Nos.: 98.80.-k,98.80.Jk,98.80.Cq

1. Introduction

After SNIa[1] and WMAP[2] observations during last decade, the discovery of the accelerated expansion of the universe is a major challenge of particle physics and cosmology. There are currently three candidates for the *Dark-Energy* which derives this accelerated expansion:

- a non-zero cosmological constant[3],
- a dynamical cosmological constant (Quintessence scalar field)[4],
- modifications of Einstein Theory of Gravity[5]

In this paper, we review shortly the main idea of three possible candidates and their cosmological phenomena. Specially we consider the interacting mechanism between dark-energy with a hot dark-matter (neutrinos). Within neutrinos probe interacting dark-energy scenario[6], we calculate Cosmic Microwave Background(CMB) radiation and Large Scale Structure(LSS) within cosmological perturbation theory. The evolution of the mass of neutrinos is determined by the quintessence scalar filed, which is responsible for the cosmic acceleration today.

2 *Yong-Yeon Keum*

2. Three possible solutions for Accelerating Universe:

Recent observations with Supernova Ia type (SNIa) and CMB radiation have provided strong evidence that we live now in an accelerating and almost flat universe. In general, one believes that the dominance of a dark-energy component with negative pressure in the present era is responsible for the universe's accelerated expansion. However there are three possible solutions to explain the accelerating universe. The Einstein Equation in General Relativity is given by the following form:

$$G_{\mu\nu} \; = \; R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \; = \; 8\pi G \, T_{\mu\nu} \; + \; \Lambda \, g_{\mu\nu}, \tag{1}$$

Here, $G_{\mu\nu}$ term contains the information of geometrical structure, the energy-momentum tensor $T_{\mu\nu}$ keeps the information of matter distributions, and the last term is so called the cosmological constant which contain the information of non-zero vacuum energy. After solve the Einstein equation, one can drive a simple relation:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{2}$$

In order to get the accelerating expansion, either cosmological constant Λ ($\omega_\Lambda = P/\rho = -1$) becomes positive or a new concept of dark-energy with the negative pressure ($\omega_\phi < -1/3$) needs to be introduced. Another solution can be given by the modification of geometrical structure which can provide a repulsive source of gravitational force. In this case, the attractive gravitational force term is dominant in early stage of universe, however at later time near the present era, repulsive term become important and drives universe to be expanded with an acceleration. Also we can consider extra-energy density contributions from bulk space in Brane-World scenario models, which can modify the Friedmann equation as $H^2 \propto \rho + \rho^{'}$. In summary, we have three different solutions for the accelerating expansion of our universe as mentioned in the introduction. Probing for the origin of accelerating universe is the most important and challenged problem in high energy physics and cosmology now. The detail explanation and many references are in a useful review on dark energy[7].

In this paper, we concentrate on the second solution using the quintessence field. In present epoch, the potential term becomes important than kinetic term, which can easily explain the negative pressure with $\omega_\phi^0 \simeq -1$. However there are many different versions of quintessence field: K-essence[8], phantom[9], quintom[10], ...etc., and to justify the origin of dark-energy from experimental observations is really a difficult job. Present updated value of the equation of states(EoS) are $\omega = -1.02 \pm 0.12$ without any supernova data[11].

3. Interacting Dark-Energy with Neutrinos:

As explained in previous section, it is really difficult to probe the origin of dark-energy when the dark-energy doesn't interact with other matters at all. Here we investigate the cosmological implication of an idea of the dark-energy interacting

with neutrinos [6,12]. For simplicity, we consider the case that dark-energy and neutrinos are coupled such that the mass of the neutrinos is a function of the scalar field which drives the late time accelerated expansion of the universe. In previous works by Fardon et al.[12] and R. Peccei[6], kinetic energy term was ignored and potential term was treated as a dynamical cosmology constant, which can be applicable for the dynamics near present epoch. However the kinetic contributions become important to descreibe cosmological perturbations in early stage of universe, which is fully considered in our analysis.

3.1. *Cosmological perturbations*

Equations for quintessence scalar field are given by

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0 \ , \tag{3}$$

$$V_{\text{eff}}(\phi) = V(\phi) + V_{\text{I}}(\phi) \ , \tag{4}$$

$$V_{\text{I}}(\phi) = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2(\phi)} f(q) \ , \tag{5}$$

$$m_\nu(\phi) = \bar{m}_i e^{\beta \frac{\phi}{M_{\text{pl}}}} \ (\text{as an example}), \tag{6}$$

where $V(\phi)$ is the potential of quintessence scalar field, $V_{\text{I}}(\phi)$ is additional potential due to the coupling to neutrino particles [12,13], and $m_\nu(\phi)$ is the mass of neutrino coupled to the scalar field. $\mathcal{H}$ is $\dot{a}/a$, where the dot represents the derivative with respect to the conformal time τ.

Energy densities of mass varying neutrino (MVN) and quintessence scalar field are described as

$$\rho_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2} f_0(q) \ , \tag{7}$$

$$3P_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{\sqrt{q^2 + a^2 m_\nu^2}} f_0(q) \ , \tag{8}$$

$$\rho_\phi = \frac{1}{2a^2}\dot{\phi}^2 + V(\phi) \ , \tag{9}$$

$$P_\phi = \frac{1}{2a^2}\dot{\phi}^2 - V(\phi) \ . \tag{10}$$

From equations (7) and (8), the equation of motion for the background energy density of neutrinos is given by

$$\dot{\rho}_\nu + 3\mathcal{H}(\rho_\nu + P_\nu) = \frac{\partial \ln m_\nu}{\partial \phi}\dot{\phi}(\rho_\nu - 3P_\nu) \ . \tag{11}$$

In our analysis, we are working in the synchronous gauge with line element:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right] \ , \tag{12}$$

For CMB anisotropies we mainly consider the scalar type perturbations. We introduce two scalar fields, $h(\boldsymbol{k}, \tau)$ and $\eta(\boldsymbol{k}, \tau)$, in k-space and write the scalar mode of

4   *Yong-Yeon Keum*

h_{ij} as a Fourier integral [22]

$$h_{ij}(\boldsymbol{x},\tau) = \int d^3k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left[\hat{\boldsymbol{k}}_i\hat{\boldsymbol{k}}_j h(\boldsymbol{k},\tau) + (\hat{\boldsymbol{k}}_i\hat{\boldsymbol{k}}_j - \frac{1}{3}\delta_{ij})6\eta(\boldsymbol{k},\tau) \right] ,$$ (13)

where $\boldsymbol{k} = k\hat{\boldsymbol{k}}$ with $\hat{k}^i\hat{k}_i = 1$.

The equation of quintessence scalar field is given by

$$\Box\phi - V_{\text{eff}}(\phi) = 0 .$$ (14)

Let us write the scalar field as a sum of background value and perturbations around it, $\phi(\boldsymbol{x},\tau) = \phi(\tau) + \delta\phi(\boldsymbol{x},\tau)$. The perturbation equation is then described as

$$\frac{1}{a^2}\ddot{\delta\phi} + \frac{2}{a^2}\mathcal{H}\dot{\delta\phi} - \frac{1}{a^2}\nabla^2(\delta\phi) + \frac{1}{2a^2}\dot{h}\dot{\phi} + \frac{d^2V}{d\phi^2}\delta\phi + \delta\left(\frac{dV_{\text{I}}}{d\phi}\right) = 0 ,$$ (15)

To describe $\delta\left(\frac{dV_{\text{I}}}{d\phi}\right)$, we shall write the distribution function of neutrinos with background distribution and perturbation around it as

$$f(x^i,\tau,q,n_j) = f_0(\tau,q)(1 + \Psi(x^i,\tau,q,n_j)) .$$ (16)

After some calculations, we finally obtain the useful equations[15]:

$$\frac{dV_{\text{I}}}{d\phi} = \frac{\partial \ln m_\nu}{\partial \phi}(\rho_\nu - 3P_\nu) ,$$ (17)

$$\delta\left(\frac{dV_{\text{I}}}{d\phi}\right) = \frac{\partial^2 \ln m_\nu}{\partial \phi^2}\delta\phi(\rho_\nu - 3P_\nu)$$

$$+\frac{\partial \ln m_\nu}{\partial \phi}(\delta\rho_\nu - 3\delta P_\nu)$$ (18)

Note that perturbation fluid variables in mass varying neutrinos are given by

$$\delta\rho_\nu = a^{-4}\int \frac{d^3q}{(2\pi)^3}\epsilon f_0(q)\Psi + a^{-4}\int \frac{d^3q}{(2\pi)^3}\frac{\partial\epsilon}{\partial\phi}\delta\phi f_0 ,$$ (19)

$$3\delta P_\nu = a^{-4}\int \frac{d^3q}{(2\pi)^3}\frac{q^2}{\epsilon}f_0(q)\Psi - a^{-4}\int \frac{d^3q}{(2\pi)^3}\frac{q^2}{\epsilon^2}\frac{\partial\epsilon}{\partial\phi}\delta\phi f_0 .$$ (20)

3.2. *Boltzmann Equation*

The Boltzmann equation is given in general,

$$\frac{Df}{D\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_C .$$ (21)

From the time component of geodesic equation [16],

$$\frac{1}{2}\frac{d}{d\tau}\left(P^0\right)^2 = -\Gamma^0_{\alpha\beta}P^\alpha P^\beta - mg^{0\nu}m_{,\nu} ,$$ (22)

and the relation $P^0 = a^{-2}\epsilon = a^{-2}\sqrt{q^2 + a^2m_\nu^2}$, we have

$$\frac{dq}{d\tau} = -\frac{1}{2}\dot{h}_{ij}qn^i n^j - a^2\frac{m}{q}\frac{\partial m}{\partial x^i}\frac{dx^i}{d\tau} .$$ (23)

Our analytic formulas in eqs.(22-23) are completely different from those of Brookfield et al.[17], since they have missed the contribution of the varying neutrino mass term. In later this term also give an important contribute in the first order perturbation of the Boltzman equation. The detail calculations will be shown in elsewhere[15].

The zeroth-order Boltzmann equation is given by

$$\frac{\partial f_0}{\partial \tau} = 0 \ . \tag{24}$$

The Fermi-Dirac distribution

$$f_0 = f_0(\epsilon) = \frac{g_s}{h_{\mathrm{P}}^3} \frac{1}{e^{\epsilon/k_{\mathrm{B}}T_0} + 1} \ , \tag{25}$$

can be a solution. Here g_s is the number of spin degrees of freedom, h_{P} and k_{B} are the Planck and the Boltzmann constants. We assume that MVNs are decoupled from the thermal bath when they are extremely relativistic so we can simply replace ϵ in the unperturbed Fermi-Dirac distribution by q. Thus we have

$$f_0 = f_0(\epsilon) = \frac{g_s}{h_{\mathrm{P}}^3} \frac{1}{e^{q/k_{\mathrm{B}}T_0} + 1} \ , \tag{26}$$

whish can also be a solution of eq.(24).

The first-order Boltzmann equation is

$$\frac{\partial \Psi}{\partial \tau} + i\frac{q}{\epsilon}(\hat{\boldsymbol{n}} \cdot \boldsymbol{k})\Psi + \left(\dot{\eta} - (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})^2 \frac{\dot{h} + 6\dot{\eta}}{2} \right) \frac{\partial \ln f_0}{\partial \ln q}$$

$$- i\frac{q}{\epsilon}(\hat{\boldsymbol{n}} \cdot \boldsymbol{k})k\delta\phi \frac{a^2 m^2}{q^2} \frac{\partial \ln m}{\partial \phi} \frac{\partial \ln f_0}{\partial \ln q} = 0 \ . \tag{27}$$

Following previous studies, we shall assume that the initial momentum dependence is axially symmetric so that Ψ depends on $\boldsymbol{q} = q\hat{\boldsymbol{n}}$ only through q and $\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}}$. With this assumption, we expand the perturbation of distribution function, Ψ, in a Legendre series,

$$\Psi(\boldsymbol{k}, \hat{\boldsymbol{n}}, q, \tau) = \sum (-i)^\ell (2\ell + 1)\Psi_\ell(\boldsymbol{k}, q, \tau)P_\ell(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}) \ . \tag{28}$$

Then we obtain the hierarchy for MVN

$$\dot{\Psi}_0 = -\frac{q}{\epsilon}k\Psi_1 + \frac{\dot{h}}{6}\frac{\partial \ln f_0}{\partial \ln q} \ , \tag{29}$$

$$\dot{\Psi}_1 = \frac{1}{3}\frac{q}{\epsilon}k\left(\Psi_0 - 2\Psi_2\right) + \kappa \ , \tag{30}$$

$$\dot{\Psi}_2 = \frac{1}{5}\frac{q}{\epsilon}k(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right) \frac{\partial \ln f_0}{\partial \ln q} \ , \tag{31}$$

$$\dot{\Psi}_\ell = \frac{q}{\epsilon}k\left(\frac{\ell}{2\ell + 1}\Psi_{\ell-1} - \frac{\ell + 1}{2\ell + 1}\Psi_{\ell+1}\right) \ . \tag{32}$$

where

$$\kappa = -\frac{1}{3}\frac{q}{\epsilon}k\frac{a^2 m^2}{q^2}\delta\phi\frac{\partial \ln m_\nu}{\partial \phi}\frac{\partial \ln f_0}{\partial \ln q} \ . \tag{33}$$

Here we used the recursion relation

$$(\ell + 1)P_{\ell+1}(\mu) = (2\ell + 1)\mu P_\ell(\mu) - \ell P_{\ell-1}(\mu) \ . \tag{34}$$

We have to solve these equations with a q-grid for every wavenumber k.

3.3. *Quintessence potentials*

To determine the evolution of scalar field which couples to neutrinos, we should specify the potential of the scalar field. A variety of quintessence effective potentials can be found in the literature. In this paper we examine three type of quintessential potentials. First we analyze what is a frequently invoked form for the effective potential of the tracker field, i.e., an inverse power law such as originally analyzed by Ratra and Peebles [30],

$$V(\phi) = M^{4+\alpha}\phi^{-\alpha} \quad \text{(Model I)} \ , \tag{35}$$

where M and α are parameters.

We will also consider a modified form of $V(\phi)$ as proposed by Brax and Martin [31] based on the condition that the quintessence fields be part of supergravity models. The potential now becomes

$$V(\phi) = M^{4+\alpha}\phi^{-\alpha}e^{3\phi^2/2m_{\mathrm{pl}}^2} \quad \text{(Model II)} \ , \tag{36}$$

where the exponential correction becomes important near the present time as $\phi \to m_{\mathrm{pl}}$. The fact that this potential has a minimum for $\phi = \sqrt{\alpha/3}m_{\mathrm{pl}}$ changes the dynamics. It causes the present value of w to evolve to a cosmological constant much quicker than for the bare power-law potential [32]. In these models the parameter M is fixed by the condition that $\Omega_\phi \approx 0.7$ at present.

We will also analyze another class of tracking potential, namely, the potential of exponential type [33]:

$$V(\phi) = M^4 e^{-\alpha\phi} \quad \text{(Model III)} \ , \tag{37}$$

This type of potential can lead to accelerating expansion provided that $\alpha < \sqrt{2}$. In figure (1), we present examples of evolution of energy densities with these three types of potentials with vanishing coupling strength to neutrinos.

3.4. *Time evolution of neutrino mass and energy density in scalar field*

For an illustration we also plot examples of evolution of energy densities for interacting case with inverse power law potential (Model I) in Fig. (2). In interacting dark energy cases, the evolution of the scalar field is determined both by its own potential and interacting term from neutrinos. When neutrinos are highly relativistic, the interaction term can be expressed as

$$\frac{\partial m_\nu}{\partial \phi}(\rho_\nu - 3P_\nu) \approx \frac{10}{7\pi^2}(am_\nu)^2\rho_{\nu_{\mathrm{massless}}} \ , \tag{38}$$
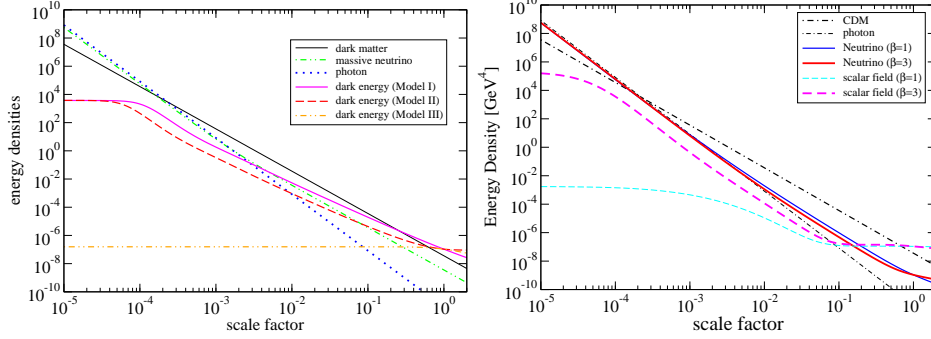
Fig. 1.   Examples of the evolution of energy density in quintessence and the background fields as indicated. Model parameters taken to plot this figure are $\alpha = 10$, 10, 1 for model I, II, III, respectively. The other parameters for the dark energy are fixed so that the energy densities in three types of dark energy should be the same at present(left-handed side figure).

Fig. 2.   Examples of the evolution of energy density in quintessence and the background fields in coupled cases with inverse power law potential (Model I). Model parameters taken to plot this figure are $\alpha = 1$, $\beta = 1$, 3 as indicated. The other parameters for the dark energy are fixed so that the energy densities in three types of dark energy should be the same at present(right-hand side figure).
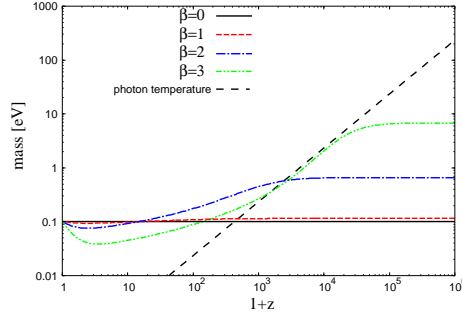

Fig. 3.   Examples of the time evolution of neutrino mass in power law potential models (Model I) with $\alpha = 1$ and $\beta = 0$ (black solid line), $\beta = 1$ (red dashed line), $\beta = 2$ (blue dash-dotted line), $\beta = 3$ (dash-dot-dotted line). The larger coupling parameter leads to the larger mass in the early universe.

where $\rho_{\nu_{\mathrm{massless}}}$ denotes the energy density of neutrinos with no mass. The term roughly scales as $\propto a^{-2}$, and therefore, it dominates deep in the radiation dominated era. However, because the motion of the scalar field driven by this interaction term is almost suppressed by the friction term, $-3\mathcal{H}\dot{\phi}$. The scalar field satisfies the slow roll condition similar to the inflation models, $-3\mathcal{H}\dot{\phi} \approx a^2 \frac{\partial m_\nu}{\partial \phi}(\rho_\nu - 3P_\nu)$. Thus, the energy density in scalar field and the mass of neutrinos is frozen there. These behaviors are clearly seen in Figs. (2) and (3).
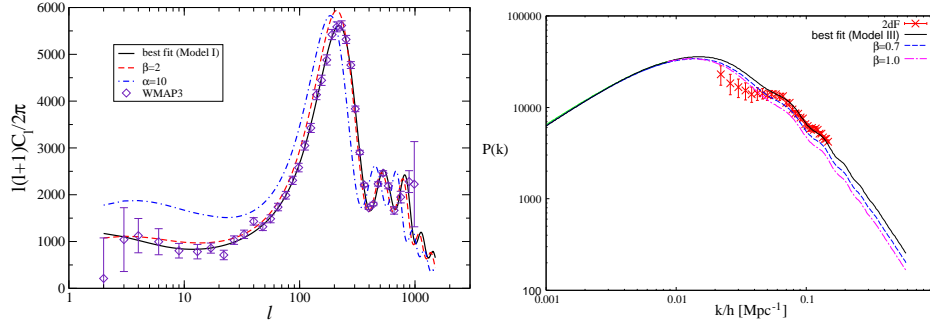
8   *Yong-Yeon Keum*

Fig. 4.   (left-hand side figure) The CMB angular power spectra for Model I. The solid line is the best fit for the model $((\alpha, \beta) = (2.97, 0.170))$, the other lines are models with different parameter value of α and β as indicated. The points are WMAP three year data.

Fig. 5.   (right-hand side figure) The CMB angular power spectra for Model III. The solid line is the best fit for the model $((\alpha, \beta) = (0.78, 0.28))$, the other lines are models with different parameter value of α and β as indicated. The points are 2dF data.

3.5. *Constrains on the MaVaNu parameters*

As was shown in the previous sections, the coupling between cosmological neutrinos and dark energy quintessence could modify the CMB and matter power spectra significantly. It is therefore possible and also important to put constraints on coupling parameters from current observations. For this purpose, we use the WMAP3 [26,27] and 2dF [28] data sets.

The flux power spectrum of the Lyman-α forest can be used to measure the matter power spectrum at small scales around $z \lesssim 3$ [34,35]. It has been shown, however, that the resultant constraint on neutrino mass can vary significantly from $\sum m_\nu < 0.2$eV to 0.4eV depending on the specific Lyman-α analysis used [36]. The complication arises because the result suffers from the systematic uncertainty regarding to the model for the intergalactic physical effects, i.e., damping wings, ionizing radiation fluctuations, galactic winds, and so on [37]. Therefore, we conservatively omit the Lyman-α forest data from our analysis.

Because there are many other cosmological parameters than the MaVaNu parameters, we follow the Markov Chain Monte Carlo(MCMC) global fit approach [29] to explore the likelihood space and marginalize over the nuisance parameters to obtain the constraint on parameter(s) we are interested in. Our parameter space consists of

$$\vec{P} \equiv (\Omega_b h^2, \Omega_c h^2, H, \tau, A_s, n_s, m_i, \alpha, \beta) \ , \qquad (39)$$

where $\omega_b h^2$ and $\Omega_c h^2$ are the baryon and CDM densities in units of critical density, H is the hubble parameter, τ is the optical depth of Compton scattering to the last scattering surface, A_s and n_s are the amplitude and spectral index of primordial
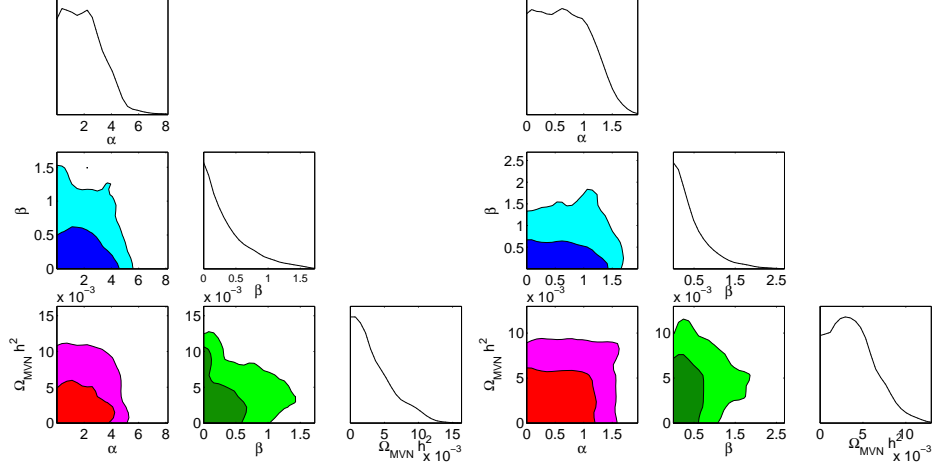
Fig. 6.   Contours of constant relative probabilities in two dimensional parameter planes for inverse power law models. Lines correspond to 68% and 95.4% confidence limits (left-hand side figure).

Fig. 7.   Same as Fig.(6), but for exponential type models (right-hand side figure).

density fluctuations, and (m_i, α, β) are the parameters of MaVaNu defined in sections 3.1 and 3.3. We have put priors on MaVaNu parameters as $\alpha > 0$, and $\beta > 0$ for simplicity and saving the computational time.

Our results are shown in Figs.(6) - (7). In these figures we do not observe the strong degeneracy between the introduced parameters. This is why one can put tight constraints on MaVaNu parameters from observations. For both models we consider, larger α leads larger w at present. Therefore large α is not allowed due to the same reason that larger w is not allowed from the current observations.

On the other hand, larger β will generally lead larger m_ν in the early universe. This means that the effect of neutrinos on the density fluctuation of matter becomes larger leading to the larger damping of the power at small scales. A complication arise because the mass of neutrinos at the transition from the ultra-relativistic regime to the non-relativistic one is not a monotonic function of β as shown in Fig.(3). Even so, the coupled neutrinos give larger decrement of small scale power, and therefore one can limit the coupling parameter from the large scale structure data.

One may wonder why we can get such a tight constraint on β, because it is naively expected that large β value should be allowed if $\Omega_\nu h^2 \sim 0$. In fact, a goodness of fit is still satisfactory with large β value when $\Omega_\nu h^2 \sim 0$, as shown in Fig.(3). However, the parameters which give us the best goodness of fit does not mean the most likely parameters in general. In our parametrization, the accepted total volume by MCMC in the parameter space where $\Omega_\nu h^2 \sim 0$ and $\beta \gtrsim 1$ was

10   *Yong-Yeon Keum*

Table 1.   Global analysis data within 1σ deviation for different types of the quintessence potential.

Quantites	Model I	Model II	Model III	WMAP-3 data (ΛCDM)
$\Omega_B\, h^2[10^2]$	2.21 ± 0.07	2.22 ± 0.07	2.21 ± 0.07	2.23 ± 0.07
$\Omega_{CDM}\, h^2[10^2]$	11.10 ± 0.62	11.10 ± 0.65	11.10 ± 0.63	12.8 ± 0.8
H_0	65.97 ± 3.61	65.37 ± 3.41	65.61 ± 3.26	72 ± 8
Z_{re}	10.87 ± 2.58	10.89 ± 2.62	11.07 ± 2.44	—
α	< 2.63	< 7.78	< 0.92	—
β	< 0.46	< 0.47	< 0.58	—
n_s	0.95 ± 0.02	0.95 ± 0.02	0.95 ± 0.02	0.958 ± 0.016
$A_s[10^{10}]$	20.66 ± 1.31	20.69 ± 1.32	20.72 ± 1.24	—
$\Omega_Q[10^2]$	68.54 ± 4.81	67.90 ± 4.47	68.22 ± 4.17	71.6 ± 5.5
$Age/Gyrs$	13.95 ± 0.20	13.97 ± 0.19	13.69 ± 0.19	13.73 ± 0.16
$\Omega_{MVN}\, h^2[10^2]$	< 0.44	< 0.48	< 0.48	$< 1.97(95\%C.L.)$
τ	0.08 ± 0.03	0.08 ± 0.03	0.09 ± 0.03	0.089 ± 0.030

small, meaning that the probability of such a parameter set is low.

We find no observational signature which favors the coupling between MaVaNu and quintessence scalar field, and obtain the upper limit on the coupling parameter within 2σ ranges as

$$\beta < 1.11,\ 1.36,\ 1.53\ , \tag{40}$$

and the present mass of neutrinos is also limited to

$$\Omega_\nu h^2_{\text{today}} < 0.0095,\ 0.0090,\ 0.0084\ , \tag{41}$$

for models I, II and III, respectively. When we apply the relation between the total sum of the neutrino masses M_ν and their contributions to the energy density of the universe: $\Omega_\nu h^2 = M_\nu/(93.14eV)$, we obtain the constraint on the total neutrino mass: $M_\nu < 0.87eV(95\%C.L.)$ in the neutrino probe dark-energy model. The total neutrino mass contributions in the power spectrum is shown in Fig 8, where we can see the significant deviation from observation data in the case of large neutrino masses.

In summary, we investigate dynamics of dark energy in mass-varying neutrinos. We show and discuss many interesting aspects of the interacting dark-energy with neutrinos scenario: (1) To explain the present cosmological observation data, we don't need to tune the coupling parameters between neutrinos and quintessence field, (2) Even with a inverse power law potential or exponential type potential which seem to be ruled out from the observation of ω value, we can receive that the apparent value of the equation of states can pushed down lesser than -1, (3) As a consequence of global fit, the cosmological neutrino mass bound beyond ΛCDM model was first obtained with the value $\sum m_\nu < 0.87\, eV(95\%CL)$.
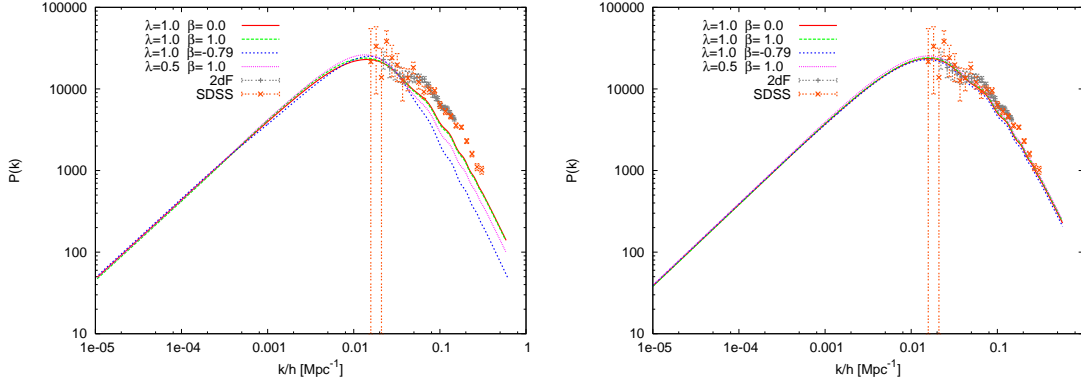
Fig. 8. Examples of the total neutrino mass contributions in power spectrum with $M_\nu = 0.9\,eV$(left-hand side graph) and with $M_\nu = 0.3\,eV$(right-hand side graph). Here the variable λ is equal to α.

Acknowledgments

References

1. Perlmtter et al.,Nature **391** (1998) 51[arXiv:astro-ph/7912212]; Riess et al., Astrophys. J. **116** (1998) 1009[arXiv:astro-ph/980520]; Perlmtter et al., ApJ **517** (1999) 565[arXiv:astro-ph/9812133].
2. C. L. Bennett et al., Astrophys. J. Suppl. Ser. **148** (2003) 1; J. L. Tonry et al.,Astrophys. J. **594** (2003) 1; M. Tegmark et al., Astrophys. J. **606** (2004) 702.
3. L. M. Krauss and M. S. Turner, Gen. Rel. Grav. **27** (1995) 1137 ; P. J. E. Peebles and B. Ratra, Reviews of Modern Physics, Vol**75** (2003) 559.
4. C. Wetterich, Nucl. Phys. B**302** (1988) 645; P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. **325** (1988) 17.
5. S. M. Carroll, M. Trodden and M. S. Turner, Phys. Rev. D**70**: 043528 (2004).
6. D. B. Kaplan, A. E. Nelson and N. Weiner, Phys. Rev. Lett. **93**:091801, (2004); R. D. Peccei, Phys. Rev. **D71**:023527 (2005).
7. E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys., **D15** 1753 (2006).
8. C. A. Picon, V. F.Mukhanov, P. J. Steinhardt, Phys. Rev. **D63**: 103510, 2001
9. R. R. Caildwell, Phys. Lett. **B 545**: 23, (2002).
10. Z. K. Guo,Y.-S. Piao, X.-M. Zhang and Y.-Z Zhang, Phys. Lett. **B 608**, 177 (2005).
11. U. Seljak, A. Slosar and P. McDonald, JCAP 0610:014 (2006).
12. R. Fardon, A. E. Nelson and N. Weiner, JCAP 0410:005, 2004; [arXiv:astro-ph/0309800].
13. X. J. Bi, P. h. Gu, X. l. Wang and X. M. Zhang, Phys. Rev. **D69**:113007 (2004); [arXiv:hep-ph/0311022].
14. J. P. Ostriker and P. Steinhardt, Science, **300**, 1909 (2003).
15. K. Ichiki and Y.-Y. Keum, "Primordial Neutrinos, Cosmological Perturbations in

Interacting Dark-Energy Models: CMB and LSS"[arXiv:astro-ph/07052134]; "Cosmological Bounds on Dark Energy-Neutrino Ineractions" (preparing draft).

16. G. W. Anderson and S. M. Carroll, arXiv:astro-ph/9711288.
17. A. W. Brookfield, C. van de Bruck, D. F. Mota, and D. Tocchini-Valentini, Phys. Rev. Lett , **96**: 061301,2006; Phys. Rev. **D73**:083515,2006.
18. D. N. Spergel *et al.*, Astrophys. J. Suppl., **148**, 175 (2003).
19. G. Hinshaw *et al.*, Astrophys. J. Suppl., **148**, 135 (2003).
20. A. Kogut *et al.*, Astrophys. J. Suppl., **148**, 161 (2003).
21. L. Verde *et al.*, Astrophys. J. Suppl., **148**, 195 (2003).
22. C. P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).
23. A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J., **538**, 473 (2000).
24. U. Seljak and M. Zaldarriago, Astrophys. J., **469**, 437 (1996).
25. W. Hu, D. Scott, N. Sugiyama, and M. White, Phys. Rev. **D52**, 5498 (1995).
26. G. Hinshaw *et al.*(WMAP collaboration), arXiv:astro-ph/0603451.
27. L. Page *et al.*(WMAP collaboration), arXiv:astro-ph/0603450.
28. S. Cole *et al.* [The 2dFGRS Collaboration], Mon. Not. Roy. Astron. Soc. **362**, 505 (2005) [arXiv:astro-ph/0501174].
29. A. Lewis and S. Bridle, Phys. Rev. **D66**, 103511 (2002).
30. B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).
31. P. Brax and J. Martin, Phys. Lett. B **468**, 40 (1999) [arXiv:astro-ph/9905040].
32. P. Brax, J. Martin and A. Riazuelo, Phys. Rev. D **62**, 103505 (2000) [arXiv:astro-ph/0005428].
33. E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D **57**, 4686 (1998) [arXiv:gr-qc/9711068].
34. P. McDonald, J. Miralda-Escude, M. Rauch, W. L. W. Sargent, T. A. Barlow, R. Cen and J. P. Ostriker, Astrophys. J. **543**, 1 (2000) [arXiv:astro-ph/9911196].
35. R. A. C. Croft *et al.*, Astrophys. J. **581**, 20 (2002) [arXiv:astro-ph/0012324].
36. A. Goobar, S. Hannestad, E. Mortsell and H. Tu, JCAP **0606**, 019 (2006) [arXiv:astro-ph/0602155].
37. P. McDonald, U. Seljak, R. Cen, P. Bode and J. P. Ostriker, Mon. Not. Roy. Astron. Soc. **360**, 1471 (2005) [arXiv:astro-ph/0407378].